**Chapter 19**

**Statistical Methods for Quality Control**

**Learning Objectives**

1. Learn about the importance of total quality, quality control, and how statistical methods can assist in the quality control process.

2. Be able to construct quality control charts and understand how they are used for statistical process control.

3. Learn about acceptance sampling procedures.

4. Know the difference between consumer’s risk and producer’s risk.

5. Know what is meant by multiple sampling plans.

6. Know the definitions of the following terms:

total quality

Six Sigma

assignable causes

common causes

control charts

upper control limit (UCL)

lower control limit (LCL)

chart

*R* chart

*p* chart

*np* chart

lot

acceptance sampling

producer’s risk

consumer’s risk

acceptance criterion

operating characteristic (OC) curve

multiple sampling plan

**Solutions:**

1. a. For *n* = 4

UCL = *μ* + 3(*σ* / ) = 12.5 + 3(.8 / ) = 13.7

LCL = *μ* – 3(*σ* / ) = 12.5 – 3(.8 / ) = 11.3

b. For *n* = 8

UCL = *μ* + 3(.8 /) = 13.35

LCL = *μ* – 3(.8 /) = 11.65

For *n* = 16

UCL = *μ* + 3(.8 /) = 13.10

LCL = *μ* – 3(.8 /) = 11.90

c. UCL and LCL become closer together as *n* increases. If the process is in control, the larger samples should have less variance and should fall closer to 12.5.

2. a. 

b. UCL = *μ* + 3(*σ* / ) = 5.42 + 3(.5 / ) = 6.09

LCL = *μ* – 3(*σ* / ) = 5.42 – 3(.5 / ) = 4.75

3. a. 

b. 

c. UCL = *p* + 3= 0.0540 + 3(0.0226) = 0.1218

LCL = *p* – 3= 0.0540 –3(0.0226) = –0.0138

Use LCL = 0

4. R Chart:

UCL = *=* 1.6(1.864) = 2.98

LCL = *=* 1.6(0.136) = 0.22

Chart:

UCL = *=* 28.5 + 0.373(1.6) = 29.10

LCL = *=* 28.5 – 0.373(1.6) = 27.90

5. a. UCL = *μ* + 3(*σ* / ) = 128.5 + 3(.4 / ) = 128.99

LCL = *μ* – 3(*σ* / ) = 128.5 – 3(.4 / ) = 128.01

b.  in control

c.  out of control

6. Process Mean = 

UCL = *μ* + 3(*σ* / ) = 20.01 + 3(*σ* / ) = 20.12

Solve for **:



7.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Sample  Number | Observations | | |  | *R*i |
| 1 | 31 | 42 | 28 | 33.67 | 14 |
| 2 | 26 | 18 | 35 | 26.33 | 17 |
| 3 | 25 | 30 | 34 | 29.67 | 9 |
| 4 | 17 | 25 | 21 | 21.00 | 8 |
| 5 | 38 | 29 | 35 | 34.00 | 9 |
| 6 | 41 | 42 | 36 | 39.67 | 6 |
| 7 | 21 | 17 | 29 | 22.33 | 12 |
| 8 | 32 | 26 | 28 | 28.67 | 6 |
| 9 | 41 | 34 | 33 | 36.00 | 8 |
| 10 | 29 | 17 | 30 | 25.33 | 13 |
| 11 | 26 | 31 | 40 | 32.33 | 14 |
| 12 | 23 | 19 | 25 | 22.33 | 6 |
| 13 | 17 | 24 | 32 | 24.33 | 15 |
| 14 | 43 | 35 | 17 | 31.67 | 26 |
| 15 | 18 | 25 | 29 | 24.00 | 11 |
| 16 | 30 | 42 | 31 | 34.33 | 12 |
| 17 | 28 | 36 | 32 | 32.00 | 8 |
| 18 | 40 | 29 | 31 | 33.33 | 11 |
| 19 | 18 | 29 | 28 | 25.00 | 11 |
| 20 | 22 | 34 | 26 | 27.33 | 12 |

= 11.4 and 

*R* Chart:

UCL = *=* 11.4(2.574) = 29.34

LCL = *=* 11.4(0) = 0

Chart:

UCL = *=* 29.17 + 1.023(11.4) = 40.8

LCL = *=* 29.17 – 1.023(11.4) = 17.5

*R* Chart:



Chart:



8. a. 

b. 

UCL = *p* + 3= 0.0470 + 3(0.0173) = 0.0989

LCL = *p* – 3= 0.0470 –3(0.0173) = –0.0049

Use LCL = 0

c. 

Process should be considered in control.

d. *p* = .047, *n* = 150

UCL = *np* + 3= 150(0.047) + 3 = 14.826

LCL = *np* – 3= 150(0.047) – 3 = –0.726

Thus, the process is out of control if more than 14 defective packages are found in a sample of 150.

e. Process should be considered to be in control since 12 defective packages were found.

f. The *np* chart may be preferred because a decision can be made by simply counting the number of defective packages.

9. a. 

b. 



Use LCL = 0.

c. 

Since .12 > UCL, the system is out of control.

d. *p* = .041 n = 100



Use LCL = 0.

e. The process is out of control since 12 > UCL.

10. 

When *p* = .02, the probability of accepting the lot is



When *p* = .06, the probability of accepting the lot is



11. a. Using binomial probabilities with *n* = 20 and *p*0 = .02.

*P* (Accept lot) = *f* (0) = .6676

Producer’s risk: ** = 1 – .6676 = .3324

b. *P* (Accept lot) = *f* (0) = .2901

Producer’s risk: ** = 1 – .2901 = .7099

12. At *p*0 = .02, the *n* = 20 and *c* = 1 plan provides

*P* (Accept lot) = *f* (0) + *f* (1) = .6676 + .2725 = .9401

Producer’s risk: ** = 1 – .9401 = .0599

At *p*0 = .06, the *n* = 20 and *c* = 1 plan provides

*P* (Accept lot) = *f* (0) + *f* (1) = .2901 + .3703 = .6604

Producer’s risk: ** = 1 – .6604 = .3396

For a given sample size, the producer’s risk decreases as the acceptance number *c* is increased.

13. a. Using binomial probabilities with *n* = 20 and *p*0 = .03.

*P*(Accept lot) = *f* (0) + *f* (1)

= .5438 + .3364 = .8802

Producer’s risk: ** = 1 – .8802 = .1198

b. With *n* = 20 and *p*1 = .15.

*P*(Accept lot) = *f* (0) + *f* (1)

= .0388 + .1368 = .1756

Consumer’s risk: ** = .1756

c. The consumer’s risk is acceptable; however, the producer’s risk associated with the *n* = 20, *c* = 1 plan is a little larger than desired.

14.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *c* | *P* (Accept) *p*0 = .05 | Producer’s Risk *α* | *P* (accept) *p*1 = .30 | Consumer’s Risk *β* |
| (*n* = 10) | 0 | .5987 | .4013 | .0282 | .0282 |
|  | 1 | .9138 | .0862 | .1493 | .1493 |
|  | 2 | .9884 | .0116 | .3828 | .3828 |
|  |  |  |  |  |  |
| (*n* = 15) | 0 | .4633 | .5367 | .0047 | .0047 |
|  | 1 | .8291 | .1709 | .0352 | .0352 |
|  | 2 | .9639 | .0361 | .1268 | .1268 |
|  | 3 | .9946 | .0054 | .2968 | .2968 |
|  |  |  |  |  |  |
| (*n* = 20) | 0 | .3585 | .6415 | .0008 | .0008 |
|  | 1 | .7359 | .2641 | .0076 | .0076 |
|  | 2 | .9246 | .0754 | .0354 | .0354 |
|  | 3 | .9842 | .0158 | .1070 | .1070 |

The plan with *n* = 15, *c* = 2 is close with ** = .0361 and ** = .1268. However, the plan with *n* = 20,

*c* = 3 is necessary to meet both requirements.

15. a. *P* (Accept) shown for *p* values below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *c* | *p*  = .01 | *p*  = .05 | *p*  = .08 | *p*  = .10 | *p*  = .15 |
| 0 | .8179 | .3585 | .1887 | .1216 | .0388 |
| 1 | .9831 | .7359 | .5169 | .3918 | .1756 |
| 2 | .9990 | .9246 | .7880 | .6770 | .4049 |

The operating characteristic curves would show the *P* (Accept) versus *p* for each value of *c*.

b. *P* (Accept)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *c* | At *p*0 = .01 | Producer’s Risk | At *p*1 = .08 | Consumer’s Risk |
| 0 | .8179 | .1821 | .1887 | .1887 |
| 1 | .9831 | .0169 | .5169 | .5169 |
| 2 | .9990 | .0010 | .7880 | .7880 |

16. a. 

b.

UCL = *μ* + 3(*σ* / ) = 95.4 + 3(.50 / ) = 96.07

LCL = *μ* – 3(*σ* / ) = 95.4 – 3(.50 / ) = 94.73

c. No; all were in control

17. a. For *n* = 10

UCL = *μ* + 3(*σ* / ) = 350 + 3(15 / ) = 364.23

LCL = *μ* – 3(*σ* / ) = 350 – 3(15 / ) = 335.77

For *n* = 20

UCL = 350 + 3(15 / ) = 360.06

LCL = 350 – 3(15 / ) = 339.94

For *n* = 30

UCL = 350 + 3(15 / ) = 358.22

LCL = 350 – 3(15 / ) = 343.78

b. Both control limits come closer to the process mean as the sample size is increased.

c. The process will be declared out of control and adjusted when the process is in control.

d. The process will be judged in control and allowed to continue when the process is out of control.

e. The controls limits for each sample size were computed using *z* = 3. Because *P*(*z* ≤ –3) = .0013, *P*(Type I error) = 2(.0013) = .0026

f. Increasing the sample size provides a more accurate estimate of the process mean and reduces the probability of making a Type II error.

18. *R* Chart:

UCL = *=* 2(2.115) = 4.23

LCL = *=* 2(0) = 0

Chart:

UCL = *=* 5.42 + 0.577(2) = 6.57

LCL = *=* 5.42 – 0.577(2) = 4.27

Estimate of Standard Deviation:



19.  = 0.665  = 95.398

Chart:

UCL = *=* 95.398 + 0.577(0.665) = 95.782

LCL = *=* 95.398 – 0.577(0.665) = 95.014

*R* Chart:

UCL = *=* 0.665(2.114) = 1.406

LCL = *=* 0.665(0) = 0

The *R* chart indicated the process variability is in control. All sample ranges are within the control limits. However, the process mean is out of control. Sample 11 (= 95.80) and Sample 17 (=94.82) fall outside the control limits.

20. = .053  = 3.082

Chart:

UCL = *=* 3.082 + 0.577(0.053) = 3.112

LCL = *=* 3.082 – 0.577(0.053) = 3.051

*R* Chart:

UCL = *=* 0.053(2.115) = 0.1121

LCL = *=* 0.053(0) = 0

All sample averages and sample ranges are within the control limits for both charts.

21. a.



Warning: Process should be checked. All points are within control limits; however, all points are also greater than the process proportion defective.

b.



Warning: Process should be checked. All points are within control limits yet the trend in points show a movement or shift toward UCL out–of–control point.

22. a. *p* = .04



UCL = *p* + 3= 0.04 + 3(0.0139) = 0.0817

LCL = *p* – 3= 0.04 – 3(0.0139) = –0.0017

Use LCL = 0

b.



For month 1 = 10/200 = 0.05. Other monthly values are .075, .03, .065, .04, and .085. Only the last month with  = 0.085 is an out-of-control situation. The seesaw (i.e. zigzag) pattern of the points is also not considered normal for an in-control process.

23. a. Use binomial probabilities with *n* = 10.

At *p*0 = .05,

*P*(Accept lot) = *f* (0) + *f* (1) + *f* (2)

= .5987 + .3151 + .0746 = .9884

Producer’s Risk: ** = 1 – .9884 = .0116

At *p*1 = .20,

*P*(Accept lot) = *f* (0) + *f* (1) + *f* (2)

= .1074 + .2684 + .3020 = .6778

Consumer’s risk: ** = .6778

b. The consumer’s risk is unacceptably high. Too many bad lots would be accepted.

c. Reducing *c* would help, but increasing the sample size appears to be the best solution.

24. a. *P* (Accept) are shown below: (Using *n* = 15)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *p* = .01 | *p* = .02 | *p* = .03 | *p* = .04 | *p* = .05 |
| *f* (0) | .8601 | .7386 | .6333 | .5421 | .4633 |
| *f* (1) | .1303 | .2261 | .2938 | .3388 | .3658 |
|  | .9904 | .9647 | .9271 | .8809 | .8291 |
|  |  |  |  |  |  |
| ** = 1 – *P* (Accept) | .0096 | .0353 | .0729 | .1191 | .1709 |

Using *p*0 = .03 since ** is close to .075. Thus, .03 is the fraction defective where the producer will tolerate a .075 probability of rejecting a good lot (only .03 defective).

b. *p* = .25

*f* (0) .0134

*f* (1) .0668

** = .0802

25. a. *P* (Accept) when *n* = 25 and *c* = 0. Use the binomial probability function with



or



|  |  |
| --- | --- |
| If | *f* (0) |
| *p* = .01 | .7778 |
| *p* = .03 | .4670 |
| *p* = .10 | .0718 |
| *p* = .20 | .0038 |

b.



c. 1 – *f* (0) = 1 – .778 = .222